

A note on the spot-futures no-arbitrage relations in a trading-production model for commodities *

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Abstract

In commodity markets, the convergence of futures towards spot prices as the time to maturity of the contracts goes to zero is usually justified by no-arbitrage arguments. In this paper we propose an alternative approach, that relies on the expected profit maximization problem of an agent producing and storing a commodity while trading in the associated futures contracts. In this framework, the relation between the spot and the futures prices holds through the well-posedness of the maximization problem. We show that the futures price can still be seen as the risk-neutral expectation of the spot price at maturity and we propose an explicit formula for the forward volatility. Moreover, we provide an heuristic analysis of the optimal solution for the production / storage / trading problem, in a Markovian setting. This approach is particularly interesting in the case of energy commodity: it remains suitable for commodities characterized by storability constraints, when standard no-arbitrage arguments can not be safely applied.

Keywords: futures contracts; no-arbitrage relation; commodity; production; storage; energy.

JEL Codes: C32; G13; Q4; Q02.

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1 Introduction

In this article we aim at explaining, through a parsimonious model, the relation between the spot and the futures prices of a commodity, that can be storable or not. The prices relation derives from the well-posedness of the simple optimization problem of an operator involved in the production and (when possible) the storage of a commodity. This producer also trades in the futures market of the commodity.

Such an approach is different from the classical no-arbitrage reasoning usually employed to explain the price relations in commodity markets. Interestingly, it remains relevant for commodities characterized by storability constraints, when standard no-arbitrage arguments can not be safely applied. This does not mean, however, that our framework is devoted only to non-storable commodities, like electricity, nor that it is focused on commodities with storable inputs (Aïd *et al.*, 2009 and 2013 [1, 3]).

Commodity futures prices fail to satisfy the no-arbitrage relation that holds for investment assets such as bonds and stocks, which can be stored at no warehousing or depreciation costs (Eydeland, 2002 [23], Chap. 4, pp. 140-43). This no-arbitrage relation states that the futures price $F(t, T)$ of a contract written on a stock, observed at date t for a delivery at T , is the spot price S_t capitalized at the interest rate r between t and T : $F(t, T) = S_t e^{r(T-t)}$.

Introducing a storage cost in the analysis is however not sufficient to depict the behavior of the prices spread in commodity markets. There is still a need to explain, first negative spreads¹, second the simultaneous presence of positive inventories and backwardated prices (see among others, Working 1933 [41]). The no-arbitrage relation holding for stocks and bonds can only be preserved by the introduction of an extra variable: the convenience yield (see Kaldor, 1940 [26] and Lautier, 2009 [29]). The latter represents an implicit revenue. The presence of such a yield, which is associated to the physical commodity but not to the futures contract, explains that the operators maintain their inventories even in the presence of negative spreads. With the introduction of such a variable, the former relation becomes $F(t, T) = S_t e^{(r-y)(T-t)}$, where y is the convenience yield net of the storage costs.

An important effort has been undertaken, in the economic and financial literature, in order to propose models that rely on storage behavior to explain and reflect the interaction between the spot and futures prices of a commodity. Within this abundant literature, our article is close to those of Brennan (1958 [14]) and Routledge *et al.* (2000 [37]). In these works, the authors develop production-storage models that connect the spot and the futures prices. Nevertheless, the models are mainly developed in order to allow for comparative statics and little information on the conditions under which no-arbitrage holds is given (this is especially true for the model of Routledge *et al.* (2000 [37])).

Apart from these models, pure financial models explicitly designed for the pricing of commodity derivatives and relying on the concept of convenience yield were also built (Lautier (2005 [28])). In our opinion, the most important reference, in this literature, is probably Schwartz (1997 [39]). In this setting however, the relation between the spot and the futures prices relies on the hypothesis that there exists a unique risk-neutral measure that one can calibrate thanks to market data.

The development of electricity markets in the last thirty years has challenged the idea that the futures price of a commodity is linked to its spot price by a convenience yield. Electricity indeed cannot be stored. Still, futures prices exhibit both contango and backwardation structures (Benth *et al.*, 2013 [11]). Many authors have pointed out that, in the case of electricity, the convenience yield may not apply. The same is true for the no-arbitrage method used in mathematical finance to obtain a risk-neutral measure (Benth *et al.*, 2003 [7]). Moreover,

¹A positive spread (a contango) arises when at date t , the futures price $F(t, T)$ is higher than the spot price S_t . A negative spread (a backwardation) is a situation where $F(t, T) < S_t$.

due to the particular nature of the futures contracts negotiated on these markets (which are basically swaps, see Frestad, 2010 [10]), even the convergence of the futures price to the spot price is an issue (Viehmann, 2011 [40]).

Since the pricing of derivatives can not rely on the concept of a convenience yield in the case of electricity, this non-storable commodity has fostered researches on how to restore a relation between the spot and the futures prices. The first approaches relied on two-date equilibrium models (Anderson and Hu, 2008 [5], Bessembinder and Lemmon, 2002 [12], Aïd *et al*, 2011 [2]), where the risk-neutral measure is extracted from the risk aversion parameters of the agents. A more complex approach consists in the extension of the market beyond the underlying asset, to include production factors (fuels) (Aïd *et al*, 2009 and 2013 [1, 3]), production constraints (Bouchard and Nguyen Huu, 2013 [13]) or gas storage levels (Douglas and Popova, 2008 [22]). In this setting, the spot price is the result of an equilibrium between production and consumption. The futures price is obtained as an expectation of the spot price.

In this article, we perform an analysis of how the futures prices can be related to the spot prices by arbitrage arguments, independently of the storability properties of the underlying asset. We do not intend, however, to provide a model especially designed for non-storable commodities. This kind of work is performed in Benth *et al* (2007 [8] and 2008 [9]), Meyers-Brandis and Tankov (2008 [31]), and Hess (2013 [24]), among others. Nor do we aim at proposing a general theory of the joint dynamics of the spot and futures prices.

Consequently, this framework could be applied to any commodity, whether it is storable or not. This could be useful for a large range of industrial companies operating in commodity futures markets, such as energy utilities, airline companies, processors of steel or of agricultural products. The profit maximization of such companies indeed relies on the spot prices as well as on the futures prices. Within our framework, once a spot price model and the market price of demand risk are chosen, there is only one futures price model that would be consistent with their optimization process.

The remaining of this article is organized as follows. In Section 2, we explain how we obtain the no-arbitrage relation through a simple model of production, storage and trading, where an operator maximizes his expected utility. This operator has no impact on the spot price. The storage cost and the production functions are supposed to be convex. The storage capacity is bounded as well as the instantaneous storage or withdrawal. Moreover, the operator has access to a derivative market where a futures written on the commodity is negotiated. The contract is traded on one maturity only and the liquidity of the futures markets is supposed to be unlimited. Naturally, the operator has no impact on the futures price.

In Section 3, inspired by previous results established in, e.g., Rogers (1994, [36]) and Ankirchner and Impeller (2005 [6]) for a pure trader, we show that the existence of a risk-neutral measure is the consequence of the finiteness of the operator's value function. Stated differently, this result means that if there were no risk-neutral measures, the operator could take advantage of his production capacity or storage facilities to get an infinite utility. In the same section, we also prove that the futures price always converges to the spot price, regardless of the storability properties of the commodity. This result is important, especially for the analysis of electricity markets. It has been indeed pointed out that the electricity futures prices predict realized spot prices rather poorly (Prevot *et al*, 2004 [34]). This study was however done on monthly contracts while the convergence issue concerns only maturities close to zero. For instance, day-ahead futures contracts quoted on the German electricity market exhibit lower discrepancy with the realized spot prices (see Viehmann, 2011 [40]).

We finally discuss in Section 4 the trading-production problem faced by the operator, with a specification of the demand dynamics. We obtain an explicit formula for the volatility of the futures contract and we relate it to the volatility of the underlying conditional demand

for the commodity. Moreover we argue that, in a Markovian setting and for an agent having a power type utility function, the optimal command for the management of storage is of a bang-bang type. Not surprisingly, the decision to store or to withdraw the commodity is based on the comparison between the spot price and the ratio between the marginal utility of one unit of storage and the marginal utility of the wealth of the operator. Lastly, in this setting the optimal trading strategy on the futures market is such that the operator holds a long position when the futures prices exhibit a positive trend.

2 The model for the individual producer

In this section we explain how we obtain the no-arbitrage relation through a simple model of production, storage and trading.

Let $(\Omega, (\mathcal{F}_t)_{t \in [0, T]}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions, i.e. (\mathcal{F}_t) is a \mathbb{P} -completed and right-continuous filtration. All processes considered in this paper are assumed to be defined in this space and adapted to this filtration.

Our agent is the producer of a commodity who also acts in the derivative market associated to this commodity. This producer is a price taker. On the physical market, he has the possibility to sell all his production, to store a part of it or to reduce his stocks. On the derivative market, he has the possibility to buy or to sell a certain amount of a unique futures contract. These choices depend on the price conditions he faces, both on the physical and on the derivative markets. In what follows, we will first focus on the physical market. Then we will expose the trading activity on the derivative market.

2.1 The profit on the physical market

On the physical market, the agent has the possibility to decide how much he produces and to manage his stocks dynamically. His instantaneous profit π_t can be written as follows:

$$\pi_t = (q_t - u_t)S_t - c(q_t) - k(X_t) \quad (2.1)$$

where:

- q_t is the production of the agent,
- u_t is the amount stored ($u_t > 0$) or withdrawn ($u_t < 0$),
- S_t is the spot price of the commodity,
- $c : \mathbb{R}_+ \rightarrow \mathbb{R}$ is the production function,
- $k : \mathbb{R}_+ \rightarrow \mathbb{R}$ is the storage function, with $k(0) = 0$,
- X_t is the storage level at time t .

Assumption 2.1. *We assume that both functions c and k are differentiable, strictly increasing, strictly convex and nonnegative.*

Moreover, we will always work under this standing assumption on the spot dynamics:

Assumption 2.2. *Let (S_t) be a bounded continuous process.*

The constraints faced by the producer on the physical market can be summarized as follows:

- the agent's production cannot exceed his capacity $\bar{q} : q_t \in [0, \bar{q}]$ for some $\bar{q} > 0$;
- instantaneous storage and withdrawal are bounded, with $u_t \in [\underline{u}, \bar{u}]$ for given thresholds $\underline{u} < 0 < \bar{u}$;
- the storage capacity itself is bounded by some $\bar{X} \geq 0$, so that adding the positivity constraint on the inventories we have $X_t \in [0, \bar{X}]$ a.s. for all $t \in [0, T]$;
- the storage dynamics is: $dX_t = u_t dt$, with $X_0 = u_0 > 0$.

In this setting, there is no uncertainty on the production of the commodity. The only source of uncertainty comes from the demand side. This hypothesis is made in order to simplify the computation and the formulas. The introduction of a stochastic production capacity driven by a Brownian motion independent of the one driving the demand would introduce market incompleteness. Besides, this hypothesis is reasonable for a large number of commodities, especially energy commodities.

2.2 The trading activity on the derivative market

We assume, for the sake of simplicity, that the interest rate is zero. Moreover, there is only one futures contract available, for a given maturity $T > 0$. The price of this contract at t is $F_t = F_t(T)$. We assume that the futures price process (F_t) is a continuous semi-martingale, adapted to the filtration (\mathcal{F}_t) . The trading portfolio on this contract is given by:

$$V_T^\theta = \int_0^T \theta_t dF_t, \quad (2.2)$$

where θ is a real-valued predictable (F_t) -integrable process. We assume that the futures market is liquid, which is standard in this context. On the empirical point of view, this is the case for the short-term maturities of commodities like crude oil and natural gas. This is however less obvious, at least up to now, for electricity markets (Aïd, 2015 [4], Chap. 2, Sect. 2.2.3).

2.3 The production-trading problem

As a commodity producer, the agent acts so as to maximize the expected utility of his terminal wealth. His utility function $U : \mathbb{R}_+ \rightarrow [-\infty, \infty[$ satisfies Inada conditions and is such that $U(x) \rightarrow \infty$ whenever $x \rightarrow \infty$. Moreover, we assume Reasonable Asymptotic Elasticity (RAE: see Kramkov and Schachermayer, 1999 [27]):

$$AE(U) := \limsup_{x \rightarrow \infty} \frac{xU'(x)}{U(x)} < 1.$$

In this setting, we propose the following production-trading problem:

$$v(r_0) := \sup_{u, q, \theta} \mathbb{E} \left[U \left(r_0 + \int_0^T \pi_t dt + V_T^\theta + \theta_{T-}(F_T - S_T) \right) \right], \quad (2.3)$$

where:

- $r_0 > 0$ is the initial wealth of the operator,

- π_t is the instantaneous profit on the physical market, expressed as a function of the quantities produced and stored, given by equation (2.1),
- V_T^θ is the trading portfolio in the futures market as given by equation (2.2),
- the term $\theta_{T-}(F_T - S_T)$ can be explained by the delivery conditions of the futures contract at expiration (see the heuristic discussion in discrete time below).

The controls (u, q, θ) have to satisfy the following additional constraints:

- constraint on the wealth of the agent to prevent infinite borrowing:

$$R_t^{x,u,q,\theta} := r_0 + \int_0^t \pi_s ds + V_t^\theta + \theta_{T-}(F_T - S_T)\mathbf{1}_{(t=T)} \geq 0, \quad t \in [0, T]. \quad (2.4)$$

- the production-storage controls $(u_t, q_t)_{t \in [0, T]}$ are adapted processes with respect to the filtration (\mathcal{F}_t) and they satisfy the constraints previously described in Paragraph 2.1.

Discrete-time heuristics. We provide some heuristics in discrete-time to justify the form of the continuous-time problem (2.3). The terminal total wealth for a trader-producer who produces, e.g., energy out of fuels, and trades in futures contracts on energy over the finite time grid $\{0, 1, \dots, T\}$ with $T \in \mathbb{N}$ can be written as follows:

$$\begin{aligned} V_T = \sum_{t=0}^{T-1} [(q_t - u_t)S_t - c(q_t) - k(X_t)] &+ \sum_{t=0}^{T-1} \theta_t(F_{t+1} - F_t) \\ &+ \theta_{T-1}F_{T-1} - h_T S_T - c(q_T) - k(X_T), \end{aligned}$$

where h_T is the quantity bought or sold at terminal date that allows to fulfill the commitment taken on the futures market at the expiration of the contract, i.e. h_T is such that

$$\theta_{T-1} = h_T + q_T - u_T,$$

so that the terminal total wealth becomes

$$V_T = \sum_{t=0}^T [(q_t - u_t)S_t - c(q_t) - k(X_t)] + \sum_{t=0}^{T-2} \theta_t(F_{t+1} - F_t) + \theta_{T-1}(F_T - S_T), \quad (2.5)$$

which constitutes the discrete-time analogue of the total wealth appearing inside the utility function in the equation (2.3).

In the next section, we will prove that a necessary condition for the problem (2.3) to be well-posed is the equality $F_T = S_T$, so that the third summand in the equation (2.5) (as well as its continuous-time counterpart in (2.3)) vanishes.

Our first objective is to deduce, from the well-posedness of the problem described by equation (2.3), the no-arbitrage condition that would link spot and futures prices, i.e. to prove that there exists some equivalent probability measure Q such that

$$F_t = \mathbb{E}_Q[S_T | \mathcal{F}_t], \quad t \in [0, T],$$

and to try to compute explicitly this futures price.

3 Existence of the optimum and spot-futures no-arbitrage relations

In this section we derive the existence and the uniqueness of an optimal solution (q^*, u^*, θ^*) for the optimization problem (2.3). At the same time, we obtain no-arbitrage relations between the spot and futures prices, as well as the convergence of the futures prices to the spot prices when the time-to-maturity goes to zero. Moreover, we show that the optimal production q^* can be computed explicitly even in this quite general framework. The way we obtain an explicit expression for the other optimal quantities, i.e. storage and trading activity (u^*, θ^*) , is discussed in Section 4.

Assumption 3.1. *Let $v(r_0) < \infty$ for some initial wealth $r_0 > 0$.*

Convergence of the futures to the spot prices and no-arbitrage relation. Our first result states that, as long as our optimization problem is well-posed, one must have a convergence of the futures price towards the spot price when the time-to-maturity tends to zero. This is true even when the underlying commodity of the contract is non storable.

Proposition 3.2. *Under Assumption 3.1, we have $F_T = S_T$.*

Proof. Assume that $\mathbb{P}(F_T \neq S_T) > 0$ and let $A = \{F_T > S_T\}$ and $B = \{F_T < S_T\}$. Consider the following trading-production strategy:

$$q = u = 0, \quad \theta_t := (\alpha \mathbf{1}_A - \beta \mathbf{1}_B) \mathbf{1}_{t=T},$$

where α and β are arbitrary positive numbers. Since A and B are \mathcal{F}_{T-} -measurable (S_t and F_t are both continuous processes), θ is clearly a predictable and F_t -integrable trading strategy. Pursuing such a strategy gives terminal wealth:

$$x + \alpha(F_T - S_T) \mathbf{1}_A + \beta(S_T - F_T) \mathbf{1}_B,$$

so that, letting $\alpha \rightarrow \infty$ and $\beta = 0$, if $\mathbb{P}(A) > 0$ or $\beta \rightarrow \infty$ and $\alpha = 0$ if $\mathbb{P}(B) > 0$ we get $v(x) = \infty$ (recall that $U(x) \rightarrow \infty$ when $x \rightarrow \infty$), which contradicts the well-posedness of our maximization problem. Thus, we can conclude that a.s. $F_T = S_T$. \square

Before proving the existence of a solution to our optimization problem, we deduce the no-arbitrage property for the futures contracts from the finiteness of $v(r_0)$. This is the content of the next proposition, which adapts arguments from Proposition 1.2 in Ankirchner and Imkeller (2005 [6]), where No Free Lunch with Vanishing Risk (henceforth NFLVR) is deduced from the well-posedness of an optimal pure investment problem. We refer to Delbaen and Schachermayer's paper (1994, [20]) for its definition as well as the proof of their celebrated version of the fundamental theorem of asset pricing. In this paper, we deduce something stronger, namely a variant of NFLVR not only for the trading portfolio but also for production and storage. To be more precise, let us redefine the NFLVR condition for our setting.

Definition 3.3. *A Free Lunch with Vanishing Risk with production and storage is a sequence of admissible plans $(q_t^n, u_t^n, \theta_t^n)$, $n \geq 1$, such that $R_T^{0,n} := R_T^{0,q^n,u^n,\theta^n}$ converges a.s. towards some nonnegative r.v. R_T^0 satisfying $\mathbb{P}(R_T^0 > 0) > 0$ and $\|(R_T^n)^-\|_\infty \rightarrow 0$ as $n \rightarrow \infty$. We will say that NFLVR with production and storage is satisfied if there are no such strategies in the model.*

Notice that since the production and storage controls are bounded, there exists a constant $M > 0$ such that $|\int_0^T \pi_t dt| \leq M$ for any admissible (q, u) giving the instantaneous profit π_t . This fact will be used in the proof of the following result.

Proposition 3.4. *Under Assumptions 2.1, 2.2, 3.1 and for all $r_0 > M$ we have that $v(r_0) < \infty$ implies NFLVR with production and storage. In particular, NFLVR for futures prices (F_t) also holds, which in turn implies that there exists at least one equivalent probability measure Q under which F_t is a local martingale.*

Proof. Suppose that NFLVR with production and storage is violated, so that we can find a sequence of terminal payoffs $R_T^n = \int_0^T \pi_t^n dt + \int_0^T \theta_t^n dF_t$ such that $R_T^n \rightarrow R_T^0$ for some nonnegative r.v. R_T^0 with $\mathbb{P}(R_T^0 > 0) > 0$ and $\varepsilon_n := \|(R_T^n)^-\|_\infty \rightarrow 0$ as $n \rightarrow \infty$. Here π^n denotes the instantaneous profit coming from a production q^n and a storage u^n . Now, take any $r_0 > 0$. There exists a $\delta > 0$ such that $\kappa := U(r_0 - M - \delta) \wedge 0 > -\infty$. Consider the sequence of strategies $(\tilde{q}^n, \tilde{u}^n, \tilde{\theta}^n)$ where

$$\tilde{q}^n = \frac{\delta}{\varepsilon_n} q^n \wedge \bar{q}, \quad \tilde{u}^n = \frac{\delta}{\varepsilon_n} u^n \wedge \bar{u} \vee (-\bar{u}), \quad \tilde{\theta}^n = \frac{\delta}{\varepsilon_n} \theta^n,$$

and let \tilde{R}^n denote the corresponding payoff. Clearly, $\tilde{R}^n \geq -\delta$ and moreover the r.v. $U(r_0 + \tilde{R}_T^n)$ is bounded from below by κ . Indeed:

$$U\left(r_0 + \int_0^T \tilde{\pi}_t^n dt + V_T^{\tilde{\theta}^n}\right) = U\left(r_0 + \int_0^T \tilde{\pi}_t^n dt + \frac{\delta}{\varepsilon_n} V_T^{\theta^n}\right) \geq U\left(r_0 - M + \frac{\delta}{\varepsilon_n}(-\varepsilon_n)\right) = \kappa > -\infty.$$

Since R_T^n converges to the nontrivial nonnegative r.v. R_T^0 and any profit is bounded by a constant M , one can find an integer n_0 and real numbers $b, c > 0$ such that $\mathbb{P}(V_T^{\theta^n} > b) > c$ for all $n \geq n_0$. Since $\kappa \leq 0$ we have

$$\begin{aligned} \liminf_n \mathbb{E}\left[U\left(r_0 + \int_0^T \tilde{\pi}_t^n dt + V_T^{\tilde{\theta}^n}\right)\right] &= \liminf_n \mathbb{E}\left[U\left(r_0 + \int_0^T \tilde{\pi}_t^n dt + \frac{\delta}{\varepsilon_n} V_T^{\theta^n}\right)\right] \\ &\geq \liminf_n \mathbb{E}\left[\kappa \mathbf{1}_{\{V_T^{\theta^n} \leq b\}} + U\left(r_0 + \int_0^T \tilde{\pi}_t^n dt + \frac{\delta}{\varepsilon_n} b\right) \mathbf{1}_{\{V_T^{\theta^n} > b\}}\right] \\ &\geq \liminf_n \mathbb{E}\left[\kappa(1 - c) + U\left(r_0 - M + \frac{\delta}{\varepsilon_n} b\right) c\right] = \infty, \end{aligned}$$

yielding that $v(r_0) = \infty$. To conclude, just recall that NFLVR with production and storage clearly implies NFLVR for trading in futures contracts, which yields in turn the existence of at least one equivalent probability measure Q for which F_t is a local martingale (see the seminal paper of Delbaen and Schachermayer, 1994 [20]). \square

Remark 3.5. The previous result implies in particular that F_t is a local Q -martingale under some equivalent probability measure Q with terminal value at time T given by the spot price S_T . If F_t was a true martingale under Q , we would recover the well-known relation

$$F_t = \mathbb{E}^Q[S_T \mid \mathcal{F}_t], \quad t \in [0, T]. \quad (3.1)$$

This would be the case if, e.g., F_t was an Itô process whose Brownian part had a square-integrable volatility process (as in the next section), or more generally if F_t was of class (D) (see, e.g. Protter 2004 [35, III.3]).

Existence and separation principle. An immediate consequence of the previous convergence result is the following separation principle, stating that solving our optimization problem is equivalent to maximize first with respect to the production control q and then with respect to the storage and trading controls (u, θ) . On the other hand, maximizing the production can also be performed in two steps. Let us denote

$$v(r_0) = \sup_{u, q, \theta} \mathbb{E}\left[U\left(r_0 + Y_T^q + Z_T^u + V_T^\theta\right)\right],$$

where we set

$$Y_T^q := \int_0^T (q_t S_t - c(q_t)) dt, \quad Z_T^u := - \int_0^T (u_t S_t + k(X_t)) dt.$$

We can solve our problem in two separate steps. First we solve $v(r_0)$ with respect to the production control q (for given u, θ); second, we solve with respect to the controls (u, θ) . Let us start from the production side.

Proposition 3.6. *Under Assumptions 2.1, 2.2, 3.1, for any given admissible investment strategy θ and storage policy u , the optimal production control q^* is given by*

$$q_t^* = (c')^{-1}(S_t) \mathbf{1}_{\{(c')^{-1}(S_t) \leq \bar{q}\}}, \quad t \in [0, T]. \quad (3.2)$$

Proof. It suffices to maximize ω -wise inside the integral in the term Y_T^q containing the production controls. Differentiating with respect to q_t for a fixed t gives $S_t - c'(q_t) = 0$ so that, taking into account the constraint $q_t \in [0, \bar{q}]$ and since c is strictly convex, we have (3.2). The proof is completed. \square

Let us denote

$$Y_T^* := Y_T^{q^*} = \int_0^T (q_t^* S_t - c(q_t^*)) dt$$

where q_t^* is given by (3.2). Now, let us consider the optimal storage/trading problem

$$v(r_0) := \sup_{u, \theta} \mathbb{E} \left[U \left(r_0 + Y_T^* + Z_T^u + V_T^\theta \right) \right]. \quad (3.3)$$

The next result establishes existence of a unique optimal storage/trading policy (u^*, θ^*) .

Proposition 3.7. *Under Assumptions 2.1, 2.2 and 3.1, there exists a unique solution (u^*, θ^*) to the problem (3.3).*

Proof. First of all, if one admits the existence of a solution, its uniqueness follows at once from the strict concavity of the utility function U . Let (u^n, θ^n) be a maximizing admissible sequence for the problem (3.3), i.e. $\mathbb{E}[U(r_0 + Y_T^* + Z_T^n + V_T^n)] \rightarrow v(r_0)$ as $n \rightarrow \infty$, where we denoted

$$Z_T^n := - \int_0^T (u_t^n S_t + k(X_t^n)) dt, \quad X_t^n := u_0 + \int_0^t u_s^n ds, \quad V_T^n := \int_0^T \theta^n dF_t.$$

We prove the compactness property of the sequences u^n and θ^n separately.

For the sequence of storage strategies u^n , we use the Komlós theorem, stating that for any sequence of r.v.'s (ξ^n) bounded in L^1 , one can extract a subsequence (ξ^{n_k}) converging a.s. in Cesaro sense to a random variable $\xi^0 \in L^1$ (see, e.g., Theorem 5.2 in Kabanov and Safarian, 2009 [25]). We apply this theorem to the sequence of processes u^n , that can be viewed as random variables defined on the product space $(\Omega \times [0, T], \mathcal{P}, d\mathbb{P}dt)$ where \mathcal{P} is the predictable σ -field. The sequence u^n is clearly in L^1 since it takes values in the interval $[-\bar{u}, \bar{u}]$. Thus, there exists a predictable process u^0 taking values in the same interval, such that the Cesaro mean sequence $\tilde{u}^n := (1/n) \sum_{j=1}^n u^j$ converges a.e. towards u^0 . Indeed it is immediate to check that the sequence \tilde{u}^n takes values in $[-\bar{u}, \bar{u}]$ as well. Moreover, the cumulated storage process along the new sequence, $\tilde{X}_t^n := \int_0^t \tilde{u}_s^n ds$, is well-defined since each \tilde{u}^n is bounded and it takes values in $[0, \bar{X}]$. By Lebesgue dominated convergence we have $\tilde{X}_t^n \rightarrow X_t^0 := \int_0^t u_s^0 ds$

a.s. for all $t \in [0, T]$. Since the function k is continuous, we have $k(\tilde{X}_t^n) \rightarrow k(X_t^0)$ a.s. for all t . Finally, thanks once more to the boundedness of the controls and to the continuity of k , we have $|\tilde{Z}_T^n| \leq C|\int_0^T S_t dt|$, which is bounded (since S_t is bounded uniformly in t). Therefore, applying the dominated convergence theorem again, we get $\tilde{Z}_T^n \rightarrow Z_T^0$ a.s. as $n \rightarrow \infty$.

As for the compactness of the sequence of trading strategies θ^n , we can work with the corresponding wealth process Cesaro mean sequence, that we denote by \tilde{V}_T^n . The admissibility property and the uniform boundedness of \tilde{Z}_T^n yields that this sequence is uniformly bounded from below by some constant. Therefore, we can apply Theorem 15.4.10 in [21], implying that there exists a convex combination $\hat{V}_T^n \in \text{conv}(\tilde{V}_T^n, \tilde{V}_T^{n+1}, \dots)$, which converges a.s. and whose limit is dominated by some $V_T^0 := \int_0^T \theta_t^0 dF_t$ for some admissible θ^0 . Moreover, applying this procedure to the Cesaro means of storage strategies \tilde{u}^n one gets another sequence of admissible storage strategies \hat{u}^n converging a.s. to the same process u^0 as \tilde{u}^n before.

To conclude the proof, we need to show that

$$v(r_0) \leq \mathbb{E}[U(x + Y_T^* + V_T^0 + Z_T^0)].$$

To do so, it suffices to use the assumption that U satisfies RAE by proceeding as in the proof of, e.g., Theorem 7.3.4 in Pham (2000 [33]). Repeating his arguments gives us the inequality above getting that (u^0, θ^0) is the optimal storage control (u^*, θ^*) . The proof of existence is now completed. \square

Remark 3.8. The fact that the trading-production problem above can be solved in successive steps does not mean that the optimal controls are independent. Only the production control q can be deduced independently from u and θ . Indeed, since the producer has no impact on the spot price, his optimal strategy is simply to equal his marginal cost of production with the spot price. Thus, no matter whether there exists a futures market or not, one would observe the same production level q . This is not the case for the optimal storage policy u and the optimal trading strategy θ : they are not independent. This fact has two consequences. First, the introduction of a futures market modifies the way storage capacities are managed. This point may be of interest for the econometric analysis of the relations between the level of the inventories and the prices. Second, as soon as the industrial process includes storage activities, the trading can not be separated from the storage without a loss of value. This should be taken into account for the organization of the trading activities in industrial companies.

4 The optimal production-trading problem

In this section we illustrate, in a simple setting, how the approach developed in the previous sections can lead to a consistent model for futures prices.

We discuss the trading-production problem faced by the operator with a specification of the demand dynamics. Then we determine the volatility of the futures contract and we relate it to the volatility of the underlying conditional demand.

4.1 The dynamics of the demand for the commodity

In order to illustrate the previous approach, we use a simple dynamics for the demand. In particular, since our goal is not to propose a model that would match all stylized facts, we do not include features such as seasonality and jumps.

In commodity markets, the spot price S_t results from the availability of the raw material on the physical market. In our model, this availability is measured through the confrontation of the total capacities of the market and the demand for the commodity, in the following way:

$$S_t = b \cdot g(\bar{C} - D_t) \cdot f(D_t), \quad (4.1)$$

with b a constant of normalization for dimension purposes, $\bar{C} > 0$ the maximum available production and storage capacities of the market (supposed constant), D_t the total exogenous demand for the commodity (which is an (\mathcal{F}_t) -adapted continuous process) and $f(D)$ the marginal cost of production and storage for a demand level D .

As there is a non negativity constraint on inventories, the spot price can jump to very high levels when the total capacities are not sufficient to fully satisfy the demand. This behavior is captured by the scarcity function g :

$$g(x) = \mathbf{1}_{x>0} \cdot \min(1/x, 1/\epsilon) + \mathbf{1}_{x<0} 1/\epsilon.$$

The effect of scarcity on commodity prices is clearly illustrated for the case of oil in Büyüksahin *et al* (2008 [15] p. 56, fig. 10). The specific form of g above has been successfully implemented in the case of electricity spot prices in Aïd *et al* (2013 [3]).

Since the production optimization problem has been solved in Proposition (3.6), it remains to treat:

$$v(x) = \sup_{u, \theta} \mathbb{E} \left[U \left(x + Y_T^* + Z_T^u + V_T^\theta \right) \right],$$

where

$$Y_T^* = \int_0^T (q_t^* S_t - c(q_t^*)) dt, \quad t \in [0, T].$$

with q_t^* as in (3.2). We recall that $Z_T^u = u_0 + \int_0^T u_t dt$ is the cumulated storage and that $V_T^\theta = \int_0^T \theta_t dF_t$ is the portfolio traded over the period $[0, T]$.

We assume that the futures price process (F_t) is an Itô process (or a diffusion to exploit the Markov property). More precisely:

Assumption 4.1. 1. Let the demand for energy D_t be mean reverting, with a long-run mean set to zero:

$$dD_t = aD_t dt + \sigma dW_t, \quad (4.2)$$

where a, σ are constants and W is a standard Brownian motion. We denote by (\mathcal{F}_t) the natural filtration generated by it and completed with the \mathbb{P} -null sets.

2. Assume that the futures price F is an Itô process fulfilling

$$dF_t = \alpha_t dt + \beta_t dW_t,$$

where α, β are some (\mathcal{F}_t) -predictable real-valued processes such that a.s.

$$\int_0^T |\alpha_t| dt + \mathbb{E} \left[\int_0^T \beta_t^2 dt \right] < \infty.$$

The integrability assumption on the volatility is here only to have the (true) martingale property of F_t and consequently the very useful formula $F_t = \mathbb{E}^Q[S_T | \mathcal{F}_t]$.

4.2 Equivalent martingale measures and forward volatility

First of all, we notice that Remark 3.5 and Assumption 4.1 imply that

$$F_t = \mathbb{E}^Q[S_T | \mathcal{F}_t], \quad t \in [0, T],$$

where Q is an equivalent martingale measure for the futures process (F_t) , which means that Q must satisfy:

$$L_t^\lambda := \frac{dQ}{d\mathbb{P}} |_{\mathcal{F}_t} = \exp \left\{ - \int_0^t \lambda_s dW_s - \frac{1}{2} \int_0^t \lambda_s^2 ds \right\},$$

where λ is a (\mathcal{F}_t) -adapted process (viewed as “market price of demand risk”) such that

- $\alpha_t - \lambda_t \beta_t = 0$ a.e. $d\mathbb{P} \otimes dt$,
- $\int_0^T \lambda_s^2 ds < \infty$,
- $\mathbb{E}[L_T^\lambda] = 1$.

At this point, in order to specify completely the dynamics of the futures price under \mathbb{P} , we need to assume a particular and tractable form for the market price of demand risk λ_t .

Assumption 4.2. *Let us assume that $\lambda_t = \lambda_0(t) + \lambda_1(t)D_t$, $t \in [0, T]$, where $\lambda_0, \lambda_1 : [0, T] \rightarrow \mathbb{R}$ are deterministic functions such that the last three properties above are satisfied.*

A consequence of this assumption is that the drift α_t of the futures price takes the form $\alpha_t = (\lambda_0(t) + \lambda_1(t)D_t)\beta_t$, which is completely determined up to the volatility β_t .

We will see in what follows that the special form of the production function defining the spot price S_T in (4.1) implies a particular functional form for the volatility of the futures price process (F_t) .

Let Q be the equivalent martingale measure corresponding to the market price of demand risk λ_t as in Assumption 4.2. Under such a measure, the demand has a dynamics characterized as follows:

$$dD_t = ((a + \lambda_1(t)\sigma)D_t + \lambda_0(t)\sigma)dt + \sigma dW_t^Q,$$

where W^Q is a standard Q -BM. Thus, the conditional distribution of D_T given D_t under Q is Gaussian with conditional mean $m_{t,T}^Q$ and variance $\Sigma_{t,T}^2$ given by

$$m_{t,T}^Q = e^{\int_t^T (a + \lambda_1(s)\sigma) ds} \left(D_t + \int_t^T e^{-\int_0^s (a + \lambda_1(u)\sigma) du} \lambda_0(s) \sigma ds \right), \quad \Sigma_{t,T}^2 = \sigma^2 \int_t^T e^{-2 \int_t^s (a + \lambda_1(u)\sigma) du} ds \quad (4.3)$$

To complete the description of our model, we set a specific shape for the marginal cost of production.

Assumption 4.3. *Let the marginal cost of production f be equal to*

$$f(d) = d^\alpha \mathbf{1}_{(0 \leq d \leq M)} + M^\alpha \mathbf{1}_{(d \geq M)}, \quad d \in \mathbb{R},$$

for some exponent $\alpha \in (0, 1)$ and some upper bound $M > 0$ such that our conditions on f are fulfilled. Moreover, let $M \geq \bar{C} - \epsilon$.

Under all these assumptions, we can express the spot price S_t as a function of the demand $S_t = \psi(D_t)$, where the function ψ is given as follows:

$$\psi(d) = b \cdot \left(\frac{d^\alpha}{\epsilon} \mathbf{1}_{(0 \leq d < \bar{C} - \epsilon)} + \frac{d^\alpha}{\bar{C} - d} \mathbf{1}_{(\bar{C} - \epsilon \leq d < M)} + \frac{M}{\bar{C} - d} \mathbf{1}_{(d \geq M)} \right). \quad (4.4)$$

Notice that the spot price S_t is always nonnegative.

The futures price at time t computed under the above measure Q is given by

$$F_t = E_t^Q[\psi(D_T)], \quad t \in [0, T],$$

where E_t^Q denotes the conditional Q -expectation given $\mathcal{F}_t = \mathcal{F}_t^D$. We denote by $h_{T,D_t}(y)$ the conditional density of D_T given D_t , that is:

$$h_{T,D_t}(y) = \frac{1}{\Sigma_{t,T} \sqrt{2\pi}} \exp \left(-\frac{(y - m_{t,T}^Q)^2}{2\Sigma_{t,T}^2} \right),$$

where the mean $m_{t,T}^Q$ and the variance $\Sigma_{t,T}^2$ are given in (4.3). We recall that the variance does not depend on D_t .

We can express the futures price F_t as a function of the demand at time t , D_t , as:

$$F_t = \varphi(t, D_t) = \int_{\mathbb{R}} \psi(y) h_{T,D_t}(y) dy.$$

A simple application of Itô's formula together with the martingale property of the futures price F_t under Q gives that the volatility of the futures price, $\beta(t, D_t)$, is given by:

$$\beta_t = \beta_t^T = \sigma \frac{\partial \varphi}{\partial d}(t, D_t).$$

If we compute explicitly the first and second derivatives of the futures price, $\varphi(t, D_t)$, with respect to the demand, we obtain the following result giving a complete specification of the parameters of the forward dynamics.

Proposition 4.4. *Under Assumptions 4.1, 4.2 and 4.3, the well-posedness of the optimal production-trading problem (2.3) implies that*

$$dF_t = \alpha_t dt + \beta_t dW_t$$

where

$$\begin{aligned} \alpha_t &= \tilde{\alpha}(t, D_t) = (\lambda_0(t) + \lambda_1(t) D_t) \beta_t, \\ \beta_t &= \tilde{\beta}(t, D_t) = \sigma \int_{\mathbb{R}} \psi(y) \frac{y - m_{t,T}^Q}{\Sigma_{t,T}^2} e^{\int_t^T (a + \lambda_1(u)\sigma) du} h_{T,D_t}(y) dy, \end{aligned}$$

for all $t \in [0, T]$. Moreover, the forward volatility $\tilde{\beta}(t, D_t)$ is increasing in the demand.

4.3 The production-trading optimization problem in a Markovian setting

In this section, we provide for the sake of completeness an informal discussion of the optimal solutions within the Markovian model determined in the previous proposition. Let us assume

that the preferences of the agent are of power type, i.e. $U(x) = x^\gamma$, $x > 0$, where $\gamma \in (0, 1)$. Recall that the problem we want to solve is the following:

$$v(x) := \sup_{(u, q, \theta) \in \mathcal{A}} \mathbb{E} \left[\left(r_0 + \int_0^T \pi_t dt + V_T^\theta \right)^\gamma \right], \quad (4.5)$$

where $r_0 > 0$ is the initial wealth. Recall that π_t is the profit rate given by:

$$\pi_t = (q_t - u_t)S_t - c(q_t) - k(X_t), \quad X_t = u_0 + \int_0^t u_s ds,$$

while $V_T^\theta = \int_0^T \theta_t dF_t$ is the gain from the self-financing portfolio traded on the futures market. \mathcal{A} denotes the set of all admissible controls (u, q, θ) . More precisely, we will say that a triplet (u, q, θ) is an admissible control if:

- $q = (q_t)_{t \in [0, T]}$ and $u = (u_t)_{t \in [0, T]}$ are adapted processes with values, respectively, in $[0, \bar{q}]$ and $[\underline{u}, \bar{u}]$;
- $\theta = (\theta_t)_{t \in [0, T]}$ is any predictable real-valued F -integrable process such that the resulting wealth is a.s. nonnegative at any time, i.e.

$$r_0 + \int_0^t \pi_s ds + V_t^\theta \geq 0, \quad t \in [0, T].$$

The relevant state variable of the problem is $Z = (R, X, D)$ where R is the wealth of the agent, i.e.

$$R_t = r_0 + \int_0^t \pi_s ds + V_t^\theta, \quad t \in [0, T].$$

The dynamics of the state variable is given by

$$\begin{aligned} dR_t &= [(q_t - u_t)\psi(D_t) - c(q_t) - k(X_t) + \alpha(t, D_t)\theta_t] dt + \beta(t, D_t)\theta_t dW_t, \\ dX_t &= u_t dt, \\ dD_t &= a D_t dt + \sigma dW_t. \end{aligned}$$

Let us introduce the value function of the optimization problem as

$$v(t, r, x, d) = \sup_{(u, q, \theta) \in \mathcal{A}_t} \mathbb{E} [(R_T)^\gamma | Z_t = (r, x, d)]$$

where \mathcal{A}_t denotes the set of all admissible controls starting at time t . The corresponding Hamilton-Jacobi-Bellman equation (hereafter HJB equation) is given by

$$-v_t - \sup_{(u, q, \theta) \in A} \mathcal{L}v = 0, \quad \text{with } A := [\underline{u}, \bar{u}] \times [0, \bar{q}] \times \mathbb{R}, \quad (4.6)$$

with terminal condition

$$v(T, r, x, d) = r^\gamma, \quad (4.7)$$

and where

$$\mathcal{L}v = uv_x + adv_d + [(q - u)\psi(d) - c(q) - k(x) + \alpha(t, d)\theta] v_r + \frac{1}{2}\sigma^2 v_{dd} + \frac{1}{2}\beta(t, d)^2 \theta^2 v_{rr}.$$

The HJB equation rewrites:

$$0 = -v_t - adv_d + k(x)v_r - \frac{1}{2}\sigma^2 v_{dd} - \sup_{(u,q,\theta) \in A} \left\{ uv_x + [(q-u)\psi(d) - c(q) + \alpha(t,d)\theta] v_r + \frac{1}{2}\beta(t,d)^2 \theta^2 v_{rr} \right\}.$$

Rearranging the terms gives

$$0 = -v_t - adv_d + k(x)v_r - \frac{1}{2}\sigma^2 v_{dd} - \sup_{(u,q,\theta) \in A} \left\{ (v_x - \psi(d)v_r)u - (c(q) - q\psi(d))v_r + \alpha(t,d)\theta v_r + \frac{1}{2}\beta(t,d)^2 \theta^2 v_{rr} \right\}. \quad (4.8)$$

We notice immediately from the above HJB equation that the optimal candidate rule for the storage management is

$$u_t^* = \underline{u} \mathbf{1}_{(\psi(D_t)v_r > v_x)} + \overline{u} \mathbf{1}_{(\psi(D_t)v_r \leq v_x)}, \quad (4.9)$$

where, to simplify the notation, we dropped the arguments (t, R_t, X_t, D_t) from the derivatives of the value functions v_r and v_x . Depending on the ratio $\eta = v_x/\psi(d)$ between the marginal utility of one unit of storage and the spot price, it is optimal either to buy and store at maximum capacity or to withdraw and sell at maximum capacity. One may have thought that this ratio should simply be compared to one. This is not the case as it has to be compared to the marginal utility of one unit of wealth, v_r . As pointed at the end of Section 3, we see that the storage control depends on the wealth and hence on the trading activities.

Furthermore, the heuristic computation above confirms the fact that the optimal control for production is to produce until the marginal cost of production equals the spot price, i.e. equation (3.2).

Finally, solving the maximisation problem in (4.8) gives the optimal control for the trading portfolio as

$$\theta_t^* = -\frac{\alpha(t, D_t)}{\beta(t, D_t)^2} \frac{v_r}{v_{rr}}(t, R_t, X_t, D_t). \quad (4.10)$$

Since it is likely that v_{rr} is negative because U is concave, one recovers the expected result that the agent holds a long position if futures prices are exhibiting a positive trend. Moreover, θ^* is similar to the Sharpe ratio, a tradeoff between the expected trend of the futures prices compared to their volatility.

Remark 4.5. To rigorously solve the optimization problem, one should prove that the Cauchy problem (4.6, 4.7) admits a unique solution with the required regularity together with a verification theorem. This could be achieved using the techniques developed in, e.g., Pham (2002 [32]) for quite a large class of multidimensional stochastic volatility models. However adapting such a method to our setting would be very technical and would go far beyond the scope of this paper.

5 Conclusion

In this paper, we develop a parsimonious model of commodity spot and futures prices, that can explain the relation between the spot and the futures prices based on arbitrage arguments. The argument does not rely on the storability of the commodity. Our setting shows that somehow, the possibility to control the production is like the ability to store, albeit for just

an instant, as is the possibility to increase or decrease the amount held in a storage capacity. From this point of view, the financial assets and the futures contracts traded on commodity markets are not so different from each others.

The existence of a risk-neutral measure is shown to be a consequence of the finiteness of the value function of an agent maximizing his utility. We also prove that the futures price always converges to the spot price, whatever the storability property of the underlying commodity is. Finally, we briefly discuss the solution of the trading-production problem faced by the agent and we show how the different controls can be separated. In particular, the optimal storage policy is impacted by the possibility of trading on the futures market.

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